



FACULTY OF ECONOMICS  
AND MANAGEMENT

## Labor Economics

(20762)

Examination Winter Semester 2012/13

**Examiner:** Prof. Dr. Andreas Knabe  
**Date:** 13.02.2013  
**The following aids may be used:** Non-programmable pocket calculators.  
Bilingual English language dictionaries without individual entries or marking.  
**Time:** 60 minutes.

Including the front page this exam contains 3 pages with 2 questions, first question containing 5 subquestions and second question containing 4 subquestions. The total amount of points to be obtained is 60. When a written explanation is asked for, please answer in short, but complete sentences and **not** just in catchwords. Remember that you should carefully explain all elements when providing graphical illustrations!

**Good Luck!**

*- Bitte wenden!*

**Question 1 (24 points): Labor Demand**

Assume a firm that uses the input factors employee-hours  $E$  and capital  $K$  to produce the output  $q$  with the production function  $f(E, K)$ . The price of labor is  $w$  and the price of capital is  $r$ .

- a) What is the difference between the short and the long run with respect to a firm's labor demand decision? (2 points)
- b) Provide a graphical representation of a firm's labor demand decision for a given level of output  $q_0$ . Use isoquants and isocost curves. (4 points)
- c) Derive the slopes of the isoquant and isocost curve. Interpret their relationship in the demand optimum. (6 points)
- d) The total effect of a change in the wage rate on long-run labor demand can be decomposed into two effects. Name both effects and describe them in your own words. Apply a suitable graph in which both effects are presented. (8 points)
- e) Name two of the four Marshallian rules of derived labor demand that were discussed in the lecture. Explain their intuition in your own words. (4 points)

**Question 2 (36 points): Schooling and Wage Earnings**

Consider the following dynamic model of human capital accumulation:

- The individual has the choice between getting educated or participate in the labor force in each point of time  $t \in [0; T]$  Note that:  $T < \infty$ .
- The discount factor is:  $r$ .
- The amount of human capital an individual possesses is:  $h(t)$ .
- The wage an individual receives when working is:  $Ah(t)$ .
- The following function indicates whether a person studies or works:

$$s(t) = \begin{cases} 1 & \text{if person studies,} \\ 0 & \text{else.} \end{cases}$$

- An efficiency parameter indicating the aptitude of the individual:  $\theta$ .
- When  $s = 1$ , human capital evolves according to:

$$\begin{aligned} \dot{h}(\theta) &= \theta h(t) \\ \Rightarrow h(\tau + x) &= h(\tau)e^{\theta x} \end{aligned}$$

- The date at which the individual starts his/her last training is denoted with  $\tau$ , the duration of the training will be  $x$  with  $\tau + x \leq T$
- The discounted present value of the individual's lifetime income stream is:

$$\Omega = \int_0^{\tau} A[1 - s(t)]h(t)e^{-rt} dt + \int_{\tau+x}^T Ah(\tau)e^{\theta x}e^{-rt} dt$$

- a) Explain the meaning of both integrals in  $\Omega$ . Derive the following optimal condition: (12 points)

$$\frac{\partial \Omega}{\partial x} = \frac{Ah(\tau)e^{\theta x}}{r} ((\theta - r)e^{-r(\tau+x)} - \theta e^{-rT}) = 0$$

- b) The optimal time at which to start schooling is  $\tau = 0$ . Use the condition in a) to derive the optimal duration of schooling. Note that  $\frac{Ah(\tau)e^{\theta x}}{r} > 0$ . (8 points)
- c) Under which condition will the individual invest in further education at  $\tau = 0$ ? (10 points)
- d) Explain in your own words the effect of the following parameters on the condition that has to hold such that the optimal  $x$  is strictly positive (that's the condition derived in c)). (6 points)

- An increase of  $\theta$ .
- A decrease of the retirement age  $T$ .
- A decrease of  $r$ .

