Original

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Examination in Mathematics II
(20.7.99)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- either script "Mathematics for Students of Economics and Management" or own lecture notes (without solved exercises)

Problems:

1. A civil engineering firm takes up a loan of 120,000 DM for buying an excavator. The interest rate is 8% annually.

   (a) In the first 3 years the firm repays 30,000 DM at the end of every year. In the 4th year nothing is repaid and the repayment at the end of the 5th year is 40,000 DM. What is the present value of the loan at the beginning of the 6th year?

   (b) How many years does the firm repay when the loan is paid off in equal installments of 12,000 DM every year (ordinary annuity).

   (c) Calculate the yearly payments required to pay off the loan after 6 years.

2. The relationship between demand x (the quantity of a good) and the price p per unit of the good is given by the demand

function

$$x(p) = \frac{450 - 10p}{p+5} \qquad 0 \le p \le 45.$$

(a) Determine zeros and extreme points of $x(p)$. Check the function for monotonicity and for concavity or convexity.

(b) Find the elasticity of demand function $x(p)$ and intervals for p where $x(p)$ is elastic or inelastic.

(c) Find the inverse demand function, i.e. the price p as a function of demand x. For which x - interval does the relation hold?

3. The profit Q for a certain agricultural product is a function of the quantity of the used sorts of chemical fertilizers r_1, r_2, r_3. Let

$$Q(r_1, r_2, r_3) = -\frac{1}{3}r_1^3 + \frac{1}{2}r_1^2 - r_2^2 - \frac{1}{2}r_3^2 + 2r_1r_2 + r_2r_3 + 2r_1 + r_2 + r_3 + 200$$

be the profit function with the domain
$r_i \ge 0,\ i = 1, 2, 3.$
Determine the quantities of the 3 used sorts of chemical fertilizers such that the profit becomes maximal. Verify the sufficient conditions, too.

4. Find the minimum of the function

$$f(x, y, z) = 2x + 2y - 2z$$

subject to

$$x^2 + 2y^2 + 3z^2 = 66.$$

Check the sufficient conditions, too.

5. Find respectively evaluate the following integrals:

(a) $\displaystyle\int (x^2 + 1) \ln x \, dx$

(b) $\displaystyle\int_0^1 \frac{x^5}{\sqrt{x^2+4}} \, dx$

(c) $\displaystyle\int_0^\infty e^{-\sqrt{x}} \, dx$