Collective Decision Making in Organizations (2687)
Final exam
Summer term 2010

Solve the 3 problems below. The bold figures indicate the maximum points per question.

The usage of pocket calculators, textbooks, lecture notes, dictionaries is not permitted. Notes on this exercise sheet will be disregarded during the grading. Give answers exclusively in your working sheets; leave a margin of 3cm.

Undecipherable scribbling will not be graded. Use the terminology and the mathematical tools presented in the lecture and the tutorial; make clear how you derive your results.

Problem 1 (18 points):

Consider a company with 71 shareholders and 100 shares in total. One large shareholder owns a block of 30 shares, while each of the other 70 small shareholders holds 1 share.

A decision by majority vote requires 51 percent support. Assume that each shareholder votes „yes" or „no" with probability 0.5 and independently from the others.

From the large shareholder's perspective, the number of „yes" votes among the 70 small shareholders is a binomial random variable denoted by $Y \sim B(70, 0.5)$. From a small shareholder's perspective, the number of „yes" votes among the other 69 small shareholders is a binomial random variable denoted by $U \sim B(69, 0.5)$. Consider the following probabilities for realizations of Y and U:

$Pr(Y \leq 20)$	$Pr(Y \leq 50)$
0.00022	0.99992

$Pr(U = 20)$	$Pr(U = 50)$
0.000196	0.000078

a)   Compute the (absolute) Penrose Power Index (PPI) for the large shareholder. **(4 points)**

b)   Compute the (absolute) Penrose Power Index (PPI) for a representative small shareholder. **(10 points)**

c)   Assume now that the large shareholder's PPI is 0.993, while one small shareholder's expected PPI is 0.0001. Compute the (relative) Banzhaf Power Index for the large shareholder and for one (representative) small shareholder. **(4 points)**

Problem 2 (12 points):

Explain, using simple voting games as example, why voting rights are a poor proxy for actual voting power. (12 points)

Problem 3 (30 points):

Consider the following preference schedule in an election with three candidates (a, b, and c). Each voter has one vote.

Number of voters:	27	24	2
1st rank	a	b	c
2nd rank	c	c	b
3rd rank	b	a	a

a)   Who wins under the simple majority rule? **(2 points)**
b)   Who wins using pairwise comparisons? **(2 points)**
c)   Who wins under the Borda count method? **(6 points)**
d)   Suppose candidate b drops out. Who wins under the Borda count method? **(6 points)**

Now consider the following preference schedule:

Number of voters:	7	8	10	4
1st rank	a	b	c	a
2nd rank	b	c	a	c
3rd rank	c	a	b	b

e)   Who wins the election under the Hare system? **(4 points)**

f)   Suppose the election in e) turns out to be invalid and, therefore, has to be repeated. As it happens, everyone votes exactly as before except for the 4 voters in the last column above. These 4 voters now decide to cast their votes according to the preference ranking c,a,b. Who wins this new election under the Hare system? **(4 points)**

g)   For both the Borda count and the Hare system, name (and briefly explain) one of the desirable criteria of voting rules that can be violated by the respective rule. **(6 points)**