Examination: 20023 – Option Pricing Theory **Winter Term 2009/2010**
Examiner: Prof. Dr. Peter Reichling
Time available: 60 minutes

Aids permitted: non-programmable pocket calculators; English dictionaries without any markings. The examination is comprised of **three** problems. All of them are to be solved. Answers must be given in **English**. **Good luck!**

Examination Questions (60 Points Total):

Problem 1 (Binomial Model – 22 Points)

A stock price quotes currently at €70 per share and can either rise by 10% or fall by 20% (per month) within the next **three** months. The (discretely compounded) risk-free interest rate is 3% p.a.

a) Using a binomial tree, show the possible stock price development. (3 points)

b) Determine the current value of an at-the-money European **put** option with a maturity of three months. Demonstrate the possible option price development with the help of a binomial tree. (9 points)

c) Assume that the put option you have considered in part b) has a maturity of one month. Calculate the theoretical price of this option. (3 points)

d) Suppose that a European put option with a strike price of €70 and a maturity of one month can be purchased at the market for €5. With the help of an arbitrage table, demonstrate how to execute an arbitrage opportunity in order to receive profit today. (7 points)

Problem 2 (Black-Scholes Model – 32 Points)

Stock X has a current price of €90 per share and a volatility of 25%. The (continuously compounded) risk-free interest rate equals 3% p.a. Suppose you hold the following portfolio consisting of options on stock X:

Option	Type	Strike price	Time to maturity	Position	Quantity
A	Call	90	200 days	Short	60
B	Call	80	150 days	Long	20

a) Within the framework of the Black-Scholes model determine the delta and the gamma of the above portfolio and interpret your result. (10 points)

b) How many stocks would you require to protect your portfolio against a small change in the price of the underlying asset? Which position (long/short) would you take? (2 points)

c) Suppose there is a put option available with a strike price of €85 that matures in 120 days. Construct a delta-neutral portfolio using put option instead of stock. Which position (long/short) and in how many puts would be required? (4 points)

d) Compute the omega of option A and interpret this measure. (6 points)

e) Explain the principle of risk-neutral valuation. (5 points)

f) Explain the application of the Black-Scholes partial differential equation in hedging. (5 points)

Problem 3 (Trading Strategies – 6 Points)

Sketch a profit profile of a bull spread strategy using put options with the strike prices K_1 and K_2, $K_2 > K_1$. In addition, construct a table which shows a payoff to each option as well as a total payoff obtained from a bull spread (consider three possible scenarios of the stock price development).

Distribution Function for the Standard Normal Distribution for Non-Negative Arguments

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767